Homework #2

I pledge my honor that I have abided by the Stevens honor system.

1.3: 2, 6, 10a, 10b, 10c, 18, 20

2.) Show that ¬(¬p) ≡ p

|  |  |  |
| --- | --- | --- |
| ***p*** | ¬p | ***¬(¬p)*** |
| ***T*** | F | ***T*** |
| ***F*** | T | ***F*** |

Therefore, p≡¬(¬p).

6.) Use a truth table to verify De Morgan’s First Law: ¬(p∧q) ≡ ¬p∨¬q

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| p | q | p∧q | ***¬(p∧q)*** | ¬p | ¬q | ***¬p∨¬q*** |
| T | T | T | ***F*** | F | F | ***F*** |
| T | F | F | ***T*** | F | T | ***T*** |
| F | T | F | ***T*** | T | F | ***T*** |
| F | F | F | ***T*** | T | T | ***T*** |

10. Show each as a tautology.

a. [¬p∧(p∨q)]→q ≡ ¬[¬p∧(p∨q)]∨q ≡ T

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| p | q | ¬p | p∨q | ¬p∧(p∨q) | ¬[¬p∧(p∨q)] | ***¬[¬p∧(p∨q)]∨q*** |
| F | F | T | F | F | T | ***T*** |
| F | T | T | T | T | F | ***T*** |
| T | F | F | T | F | T | ***T*** |
| T | T | F | T | F | T | ***T*** |

[¬p ∧ (p ∨ q)] → q

≡ ¬[¬p ∧ (p ∨ q)] ∨ q implicit exchange

≡ p ∨ ¬(p ∨ q) ∨ q double negation law

≡ p ∨ ¬p ∧¬q ∨ q De Morgan’s Law/associative

≡ p ∨ ¬p ∧ q ∨ ¬q commutative property

≡ T ∧ T Negation law

≡ T Idempotent

b. [(p → q) ∧ (q → r)] → (p → r)

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| p | q | r | ¬p | ¬q | ¬p∨¬q | ¬q∨r | (¬p∨q)∧(¬q∨r) | ¬[(¬p∨q)∧(¬q∨r)] | ¬p∨r | ***¬[(¬p∨q)∧(¬q∨r)]∨(¬p∨r)*** |
| F | F | F | T | T | T | T | T | F | T | ***T*** |
| F | F | T | T | T | T | T | T | F | T | ***T*** |
| F | T | F | T | F | T | F | F | T | T | ***T*** |
| F | T | T | T | F | T | T | T | F | T | ***T*** |
| T | F | F | F | T | F | T | F | T | F | ***T*** |
| T | F | T | F | T | F | T | F | T | T | ***T*** |
| T | T | F | F | F | T | F | F | T | F | ***T*** |
| T | T | T | F | F | T | T | T | F | T | ***T*** |

[(p → q) ∧ (q → r)] → (p → r)

≡ [(¬p ∨ q) ∧ (¬q ∨ r)] → (¬p ∨ r) implicit exchange

≡ ¬[(¬p ∨ q) ∧ (¬q ∨ r)] ∨ (¬p ∨ r) implicit exchange

≡ ¬(¬p ∨ q) ∨ ¬(¬q ∨ r) ∨ (¬p ∨ r) De Morgan’s Law

≡ (p ∧ ¬q) ∨ (q ∧ ¬r) ∨ (¬p ∨ r) De Morgan’s Law

≡ p ∧ (¬q ∨ q) ∧ ¬r ∨ (¬ p ∨ r) Associative property

≡ p ∧ T ∨ ¬r ∨ (¬p ∨ r) Negation Law

≡ p ∧ ¬r ∨ T ∨ (¬p ∨ r) Commutative property

≡ p ∧ ¬r ∨ (¬p ∨ r) ∨ T Commutative property

≡ p ∨ ¬p ∧ r ∨ ¬r ∨ T Commutative property

≡ T ∧ T ∨ T Negation Law

≡ T Idempotent

c. [p ∧ (p → q)] → q

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| p | q | ¬p | ¬p∨q | p∧(¬p∨q) | ¬(p∧(¬p∨q)) | ***¬[p∧(¬p∨q)]∨q*** |
| T | T | F | T | T | F | ***T*** |
| T | F | F | F | F | T | ***T*** |
| F | T | T | T | F | T | ***T*** |
| F | F | T | T | F | T | ***T*** |

[p ∧ (p → q)] → q

≡ ¬[p ∧ (¬p ∨ q)] ∨ q Implicit exchange

≡ ¬[(p ∧ ¬p) ∨ q] ∨ q Associative property

≡ ¬(p ∧ ¬p) ∧ ¬q ∨ q De Morgan’s Law

≡ ¬p ∨ p ∧ ¬q ∨ q De Morgan’s Law

≡ T ∧ T Negation Law

≡ T Idempotent

18.) Show that p → q and ¬q → ¬p are logically equivalent.

p → q

≡ ¬p ∨ q Implicit exchange

≡ ¬p ∨ ¬¬q Commutative property

≡ ¬¬q ∨ ¬p Commutative property

≡ ¬q → ¬p Implicit exchange

20.) Show that ¬(p ⊕ q) ≡ p ↔ q

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| p | q | p⊕q | ***¬(p⊕q)*** | p→q | q→p | ***(p→q)∧(q→p)*** |
| F | F | F | ***T*** | T | T | ***T*** |
| F | T | T | ***F*** | T | F | ***F*** |
| T | F | T | ***F*** | F | T | ***F*** |
| T | T | F | ***T*** | T | T | ***T*** |

1.4: 6, 8, 10, 12, 28

6.) Let N(x) be “x has visited ND” , domain consists of students in school. Express in English.

a. ∃x N(x): There is some student at school that has visited ND.

b. ∀x N(x): Every student at school has visited ND.

c. ¬∃x N(x): No student at school has visited ND.

d. ∃x ¬N(x): There is some student at school who has not visited ND.

e. ¬∀x N(x): Not all students at school have visited ND.

f. ∀x ¬N(x): None of the students at school have visited ND.

8.)R(x) is “x is a rabbit” and H(x) is “x hops”, domain consists of all animals. Express in English.

a. ∀x [R(x) →H(x)]: Every animal, that is a rabbit, hops.

b. ∀x [R(x) ∧ H(x)]: All animals are rabbits, and they hop.

c. ∃x [R(x) →H(x)]: There is an animal that hops if it is a rabbit.

d. ∃x [R(x) ∧ H(x)]: There is an animal that is a rabbit and hops.

10.) C(x) is “x has a cat”, D(x) is “x has a dog”, F(x) is “x has a ferret”, domain is students in your class.

a. A student in your class has a cat, a dog, and a ferret: ∃x [C(x) ∧ D(x) ∧ F(x)]

b. All students in your class have a cat, a dog, or a ferret: ∀x [C(x) ∨ D(x) ∨ F(x)]

c. Some student in your class has a cat, and a ferret, but not a dog: ∃x [C(x) ∧F(x) ∧ ¬D(x)]

d. No student in your class has a cat, a dog, and a ferret: ¬∃x [C(x) ∧ D(x) ∧ F(x)]

e. For each C(x), D(x), and F(x), there is a student who has one: ∃x C(x) ∧ ∃x D(x) ∧ ∃x F(x)

12.) Q(x) is “x+1>2x”. What are the truth-values, where domain is all ints?

a. Q(0) = T

b. Q(-1) = T

c. Q(1) = F

d.∃x Q(x) = T

e.∀x Q(x) = F

f. ∃x ¬Q(x) = T

g.∀x ¬Q(x) = F

28.) Cor(x) is “x is in the correct place”, Tool(x) is “x is a tool”, EC(x) is “x is in excellent condition”.

a. Something is not in the correct place: ∃x ¬Cor(x)

b. All tools are in the correct place and are in excellent condition:∀x [Tool(x) → Cor(x) ∧ EC(x)]

c. Everything is in the correct place and are in excellent condition: ∀x [Cor(x) ∧ EC(x)]

d. Nothing is in the correct place and is in excellent condition: ¬∃x [Cor(x) ∧ EC(x)]

e. One of your tools is not in the correct place, but it’s in excellent condition: ∃x (Tool(x)→¬Cor(x)∧EC(x)